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# Influence of Dielectric Losses on the Shift of the Fundamental Frequencies of Thickness Mode Piezoelectric Ceramic Resonators

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***Abstract***

*The influence of dielectric losses on the shifts of the characteristic frequencies* $f_s$ *and* $f_p$*, defined as the frequencies of maximum electrical conductance and resistance respectively, is investigated at the fundamental resonance bands of thickness extensional piezoelectric ceramic resonators. The more general expression for the complex electrical input impedance is used as a reference instead of the usual Butterworth–Van Dyke equivalent circuit. The concept of normalised electrical impedance of the lossy resonator is introduced and applied to the computation of the departs of the characteristic frequencies* $f_s$ *and* $f_p$*, from the associated lossless critical frequencies* $f_1$ *and* $f_2$*, as a function of* $k_t$*, tan* $\delta_m$*, and tan* $\delta_e$*. It is shown that, according to the adopted model, the effects of dielectric losses on the critical frequencies are different than those produced by mechanical losses: on the one hand, intrinsic mechanical losses increase* $f_s$ *and lower* $f_p$*; on the other hand, dielectric losses lower* $f_s$ *and augment the decrease of* $f_p$*. The frequency displacements have been computed over a wide range of the fundamentals parameters, which include typical values of ceramic materials used for ultrasonic imaging applications.* 

*Keywords*: piezoelectric resonances, piezoelectric properties, electrical properties, dielectric properties, actuators.

## 1 Introduction

The relevant parameters of a piezoelectric resonator can be determined from measurements of its electrical input impedance as a function of frequency. The complex electrical impedance curves and the associated critical frequencies are the basis of this piezoelectric characterisation by the resonance method.

Intrinsic losses in the piezoelectric media have a notable influence on the electrical impedance and associated characteristic frequencies of resonance bands. While these losses have a first order influence on the values of the electrical impedance, they have a second order effect on the characteristic frequencies of the resonance. Nevertheless, some of these characteristic frequencies play a key role on the characterisation of piezoelectric materials by the resonance method.

The effects of losses on the fundamental frequencies of thickness mode piezoelectric resonators have been previously studied, considering (i) the Butterworth–Van Dyke equivalent circuit as a reference,[1] (ii) assuming a complex electromechanical coupling coefficient and exploring the behaviour in the complex frequency plane,[2,3] and (iii) using a complex elastic stiffness constant approach to evaluate the effect of intrinsic mechanical losses.[4,5]

The present work investigates the influence of both intrinsic mechanical and dielectric losses on the shifts of the characteristic frequencies $f_s$ and $f_p$ at the fundamental resonance band of thickness extensional piezoelectric ceramic resonators. Instead of the usual Butterworth–Van Dyke equivalent circuit, the general expression for the complex electrical input impedance is used as a reference. The frequencies $f_s$ and $f_p$ are defined as the frequency of maximum electrical conductance, and the frequency of maximum electrical resistance, respectively, and this definition does not depend on the adopted transduction model. These frequencies are of special interest since they are normally used for the evaluation of material constants, according to

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the current ANSI/IEEE Standard on Piezoelectricity, instead of the ideal lossless lower and upper critical frequencies[6] $f_1$ and $f_2$.

## 2 Electrical Impedance of Thickness Extensional Piezoelectric Resonators with Intrinsic Mechanical and Dielectric Losses

The thickness excitation of a piezoelectric plate made of ferroelectric ceramic, or piezoelectric ceramic-polymer composite, polarised in the thickness direction, with a pair of electrodes over the major surfaces, and lateral dimensions much greater than thickness, produces a thickness extensional mode of vibration. These type of thickness-mode resonators are frequently used in ultrasonic imaging applications.

The general expression of the electrical input impedance of a lossless piezoelectric resonator driven in a thickness extensional pure mode of vibration is given by[6,7]

$$Z(\omega) = \frac{1}{j\omega C_0^s}\left(1 - k_t^2 \frac{\tan(\gamma/2)}{(\gamma/2)}\right) \quad (1)$$

where $\omega$ is the angular frequency, $C_0^s$ the clamped capacitance of the piezoelectric disk with electrode area $A$ and thickness $t$; $k_t$ is the electromechanical coupling coefficient, being

$$C_0^S = A\varepsilon_{33}^S/t\ ; \quad (2.a)$$

$$k_t = h_{33}(\varepsilon_{33}^s/c_{33}^D)^{1/2}\ ; \quad (2.b)$$

$$\gamma = \omega t/(c_{33}^D/\rho)^{1/2}\ ; \quad (2.c)$$

where $c_{33}^D, h_{33}$, and $\varepsilon_{33}^S$, are elastic, piezoelectric, and dielectric constants, respectively, and $\rho$ is the mass density. Equation (1) is the more general expression for the electrical impedance under the assumptions of linear theory of piezoelectricity, one-dimensional mode of vibration, and negligible losses.

For the mode of vibration under consideration, mechanical loss can be introduced by means of a complex elastic constant $(c_{33}^D)^*$, and can be interpreted as a phase lag (phase angle $\delta_m$) of strain behind the stress.

$$(c_{33}^D)* = c_{33}^D(1 + j\tan\delta_m)$$

Dielectric loss is conventionally introduced by means of a complex dielectric constant, $(\varepsilon_{33}^S)^*$, with $\tan\delta_e$ being the clamped dielectric loss tangent:

$$(\varepsilon_{33}^S)* = \varepsilon_{33}^S(1 - j\tan\delta_e)$$

From this description, the effect of mechanical and dielectric losses in the characteristic frequencies, can be discussed independently. Inclusion of the complex coefficients in eqn (1) yields

$$Z(\omega) = \frac{1}{j\omega C_0^S(1 - j\tan\delta_e)}\left[1 - k_t^2 \frac{(1 - j\tan\delta_e)}{(1 + j\tan\delta_m)^{1/2}} \frac{\tan\left((\gamma/2)(1 + j\tan\delta_m)^{-1/2}\right)}{(\gamma/2)}\right] \quad (3)$$

It is assumed, as it as usual in piezoelectric characterisation, that the parameters are constants and independent of signal level (linear theory). The loss tangents are considered independent of frequency, in the analysed short frequency interval $(f_1 - f_2)$ around the fundamental resonance, and the piezoelectric constant $h_{33}$ is assumed to be real. Frequency dependence of loss parameters can be introduced between different overtone resonances.

## 3 Shift in the Location of the Fundamental Frequencies: Methodology and Results

According to eqn (3), the particular variation of complex electrical impedance with frequency depends on only five parameters: $C_0^S, k_t, \gamma, \tan\delta_m$, and $\tan\delta_e$. As presented previously,[4] the impedance values can be expressed as a function of the relative frequency $f/f_0$, and can be normalised multiplying by $2\pi f_0 C_0^S$. The frequency $f_0$ is defined here as the nominal frequency, that is the frequency of the fundamental mechanical resonance in the absence of losses, $[f_0 = (c_{33}^D/\rho)^{1/2}/(2t)]$. It should be noted that $f_0$ is the same that the lossless upper critical frequency[6] $f_2, [f_0 = f_2]$, and that $\gamma = \pi f/f_0$. We denote the resulting impedance values as the normalised electrical input impedance. Therefore, three independent variables characterise the electrical behaviour of these resonators, namely: the intrinsic electromechanical coupling coefficient $k_t$, the intrinsic mechanical losses expressed by means of $\tan\delta_m$, and the intrinsic dielectric losses expressed by means of $\tan\delta_e$.

Figure 1 illustrates the normalised electrical input impedance, around the fundamental resonance, of three types of thickness mode resonators with different active materials: (i) PZT, (ii) PMN type piezoelectric ceramics, and (iii) a conventional ceramic-polymer piezoelectric composite. Table 1 summarises a set of typical values of the parameters $k_t$, $\tan\delta_m$ and $\tan\delta_e$, corresponding to these three types of piezoceramic materials (frequently

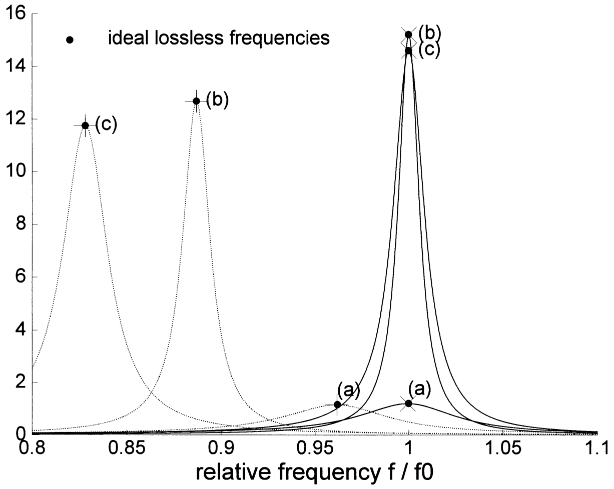


**Fig. 1.** Normalised electrical input conductance $G$ (dotted lines) and resistance $R$ (continuous lines), around the fundamental resonance, of three thickness mode piezoelectric resonators with different active materials, (a), (b), and (c) as defined in Table 1. The same scale in the ordinate axis is valid in this case for $G$ and $R$. The symbols + and × show respectively the location of the frequencies $f_s$ and $f_p$ of lossy resonators.

used for ultrasonic imaging applications), which are used in the computation of the impedance values in Fig. 1. Both, the real part of the complex impedance $R$, and real part of the complex admittance $G$, as well as the corresponding ideal lossless characteristic frequencies $f_1/f_0$, and $f_2/f_0 = 1$, are shown for each type of material. Table 1 also summarises, the relative lower and upper critical frequencies $f_1/f_0$, and $f_2/f_0$, which depend on only $k_t$, as well as the relative characteristic frequencies of the lossy resonators $f_s/f_0$, and $f_p/f_0$, which have been computed from eqns (1) and (3) respectively. The computed frequency displacements $(\delta f_s) = 100(f_s - f_1)/f_1$, and $(\delta f_p) = 100(f_2 - f_1)/f_2$, are also included. It can be noticed that for these kind of piezoelectric materials, the frequency displacements produced by intrinsic losses are very small. The last column in Table 1 shows the relative error in the determination of the electromechanical coupling coefficient (% kt) from the conventional resonance-antiresonance frequency spacing $(f_p - f_s)$, according to the usual expression.

$$k_t^2 = (\pi f_S/2f_P)\tan(\pi(f_P - f_S)/2f_P)$$

It should be noted that for the typical values of the parameters of these piezoelectric materials, the errors are very small and the coupling factor $k_t$ can be determined from the relative frequency spacing, without the use of iterative computer fitting methods of the theoretical and experimental impedance/admittance curves. These results are in accordance with Ref.4, although dielectric losses were neglected in that work.

A specific computer program has been written in order to analyse the changes of the characteristic frequencies over a wide range of variation of the resonator parameters $k_t$, tan $\delta_m$, and tan $\delta_e$. In each case, the complex electrical impedance of the lossy resonator is computed around the fundamental resonance from eqn (3). For each particular value of the intrinsic coupling coefficient $k_t$, the characteristic frequencies $f_1$ and $f_2$ are obtained in the limit when tan $\delta_m \to 0$, and tan de $\delta_e \to 0$. Then, for each particular value of the mechanical loss tangent tan $\delta_m$, and the dielectric loss tangent tan $\delta_e$, the frequencies $f_s$ and $f_p$ are determined from the position of the maximum conductance $G$ and maximum resistance $R$ respectively.

Figure 2(a) shows the evolution of the relative resonance frequency of the lossy resonators $f_s/f_0$, as a function of tan $\delta_m$, computed using the previous procedure, in the case of $k_t = 0{\cdot}5$. Four different cases of dielectric losses have been considered: (a) $\tan\delta_e = 0{\cdot}001$, continuous line; (b) $\tan\delta_e = 0{\cdot}01$, dashed line; (c) $\tan\delta_e = 0{\cdot}03$, dotted line; and (d) $\tan\delta_e = 0{\cdot}1$, dash-dotted line. The ordinate axis represents increasing values of mechanical losses. For each particular combination of $\tan\delta_m$ and $\tan\delta_e$, the abscissa gives the resonance frequency $f_s/f_0$. The asterisk in the x-axis shows the position of the lower critical frequency $f_1/f_0$ (resonance frequency in the absence of losses). As can be seen, the relative resonance frequency $f_s/f_0$ decreases as dielectric losses are increased (moving from continuous to dash-dotted lines, for a fixed value of $\tan\delta_m$). The effect of intrinsic mechanical losses is to increase the resonance frequency, as can be clearly appreciated in the first 3 curves (continuous, dashed and dotted). The combined effect of high dielectric losses with mechanical losses can be appreciated in the dash-dotted line ($\tan\delta_e = 0{\cdot}1$) .

Figure 2(b) is analogous to Fig. 2(a) but corresponds to the relative antiresonance frequency $f_p/f_0$. This figure clearly shows how the effect of intrinsic mechanical losses is to lower the antiresonance

**Table 1.** Input parameters, relative characteristic frequencies, and frequency displacements for three types of piezoceramic active materials

| *Type* | $k_t$ | *tan* $\delta_m$ | *tan* $\delta_e$ | $f_1/f_0$ | $f_s/f_0$ | $f_2/f_0$ | $f_p/f_0$ | $\delta f_s$ | $\delta f_p$ | $\%k_t$ |
|---|---|---|---|---|---|---|---|---|---|---|
| (a) PMN | 0·3 | 0·0625 | 0·03 | 0·96213 | 0·96167 | 1·00 | 0·99950 | −0·046 | 0·050 | 0·028 |
| (b) PZT | 0·5 | 0·0133 | 0·01 | 0·88697 | 0·88692 | 1·00 | 0·99997 | −0·005 | 0·002 | −0·008 |
| (c) Composite | 0·6 | 0·0200 | 0·01 | 0·82811 | 0·82807 | 1·00 | 0·99995 | −0·006 | 0·005 | 0·000 |

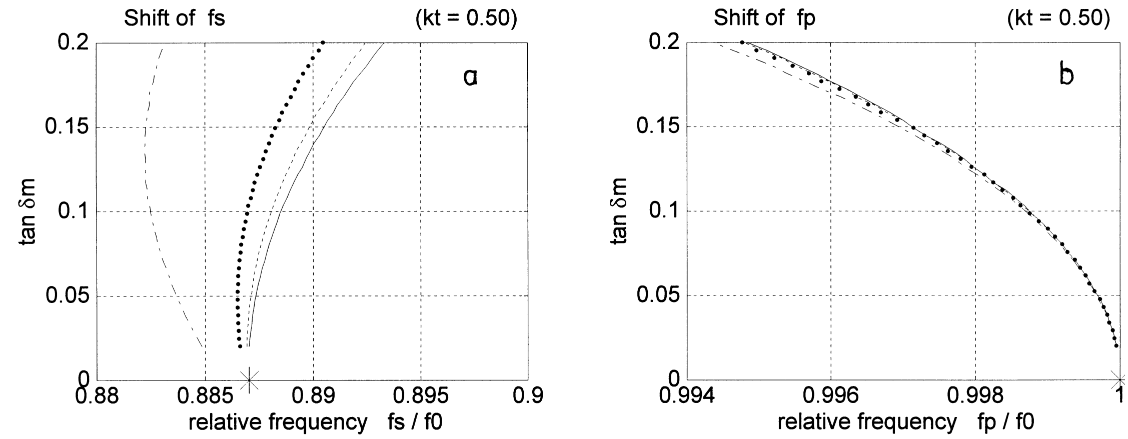


**Fig. 2.** Evolution of the characteristic frequencies ($x$-axis) with tan $\delta_m$ ($y$-axis), when $k_t = 0{\cdot}5$. Continuous line, $\tan\delta_e = 0{\cdot}001$; dashed line, $\tan\delta_e = 0{\cdot}01$; dotted line, $\tan\delta_e = 0{\cdot}03$; and dash-dotted line, $\tan\delta_e = 0{\cdot}1$. (a) $f_s/f_0$, and (b) $f_p/f_0$.

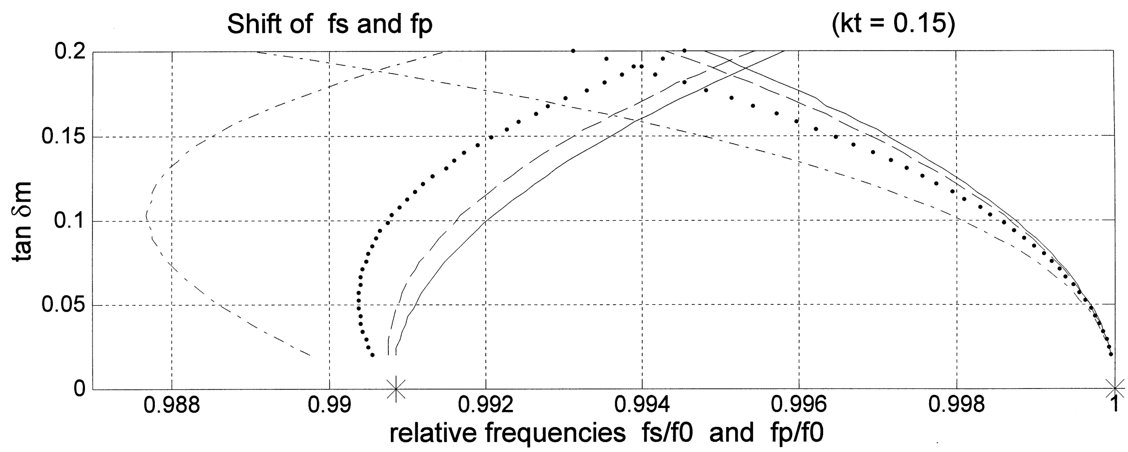


**Fig. 3.** Analogous to Fig. 2, but corresponds to the case $k_t = 0{\cdot}15$. Both relative frequencies $f_s/f_0$, and $f_p/f_0$ are displayed together in the same $x$-axis.

frequency. For a given value of $\tan\delta_m$, as dielectric losses are increased, $f_p/f_0$ are further decreased.

Figure 3 corresponds to the case of $k_t = 0{\cdot}15$, with both relative frequencies $f_s/f_0$, and $f_p/f_0$ displayed together in the x-axis. It can be noticed how the shift of the characteristic frequency $f_p/f_0$ increases for weak electromechanical coupling. It should be highlighted the presence of frequency crossings such that $f_s > f_p$, for high values of $\tan\delta_m$. In these extreme cases, with high losses and low coupling factor, the relative frequency spacing is negative, and an erroneous negative value of the squared coupling factor would be obtained if the standard procedure for measuring $k_t$ were applied.

## 4 Conclusions

The concept of normalised electrical impedance of the lossy resonator has been introduced and applied to the computation of the departs of the characteristic frequencies $f_s$ and $f_p$, from the associated critical frequencies $f_1$ and $f_2$, as a function of $k_t$, $\tan\delta_m$, and $\tan\delta_e$. It has been shown that according to the adopted exact one-dimensional model, the effects of dielectric losses on the characteristic frequencies are different than those produced by intrinsic mechanical losses, and therefore they can not be simply coupled together in a complex electromechanical coupling coefficient. Intrinsic mechanical losses increase $f_s$ and lower $f_p$. The frequency displacement $(f_2 - f_p)$ increases for highly attenuating piezoelectric materials with weak electromechanical coupling. The frequency displacement $(f_s - f_1)$ increases with both $\tan\delta_m$ and $k_t$. Dielectric losses lower $f_s$ and augment the displacement of $f_p$ produced by mechanical losses. The combined effect of both intrinsic mechanical and dielectric losses on the relative frequency spacing $(f_p - f_s)/f_0$, which is the basic measure for the determination of the electromechanical coupling coefficient $k_t$, depends on the particular combination of values of the coupling factor and loss tangents, as can be appreciated in Figs 2 and 3.

For typical values of the coupling factor and loss parameters corresponding to piezoceramics of PZT and PMN types, and conventional ceramic-polymer piezoelectric composites, the frequency displacements are small (Table 1), and the expressions of the IEEE Standards used for the determination of the thickness coupling coefficient $k_t$, and the thickness frequency constant $N_t$ are accurate within a 0·05%, so that there is no need for the use of iterative computer methods. This is not the case of piezoelectric polymers, with a low value of the coupling factor and high values of the mechanical and dielectric losses, which will be analysed with the present methodology in a further work.

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